Four-photon *W* state using two-crystal geometry parametric down-conversion

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An experimental scheme for producing a four-photon entangled *W* state directly from the parametric down-conversion of a two-crystal geometry is proposed. In our scheme, only linear optical elements and photon detectors are required. Sophisticated and fragile interferometric setups are unnecessary.

Entangled states of two or more particles not only play a central role in discussions of nonlocal quantum correlations [1], but also form the basis of quantum information, such as quantum cryptography [2], dense coding [3], teleportation [4], and quantum computation [5]. Recently, there has been much interest in the investigation of multiparticle entangled states. It was shown that there are two different kinds of genuine tripartite entanglement [6]. One is the Greenberger-Horne-Zeilinger (GHZ) state [7]

$$|\text{GHZ}^{(3)}\rangle = \frac{1}{\sqrt{2}}(|HHH\rangle + |VVV\rangle). \qquad (1)$$

The other is the *W* state

$$|W^{(3)}\rangle = \frac{1}{\sqrt{3}}(|HHV\rangle + |HVH\rangle + |VHH\rangle). \qquad (2)$$

The GHZ state and *W* state are inequivalent, because they cannot be converted to each other under stochastic local operations and classical communication (SLOCC). In the case of four particles, it was shown that there are nine different classes of states not equivalent under SLOCC [8] and the four-particle *W* state belongs to one of them. It is interesting that entanglement of the four-particle *W* state is highly robust against the loss of one or two qubits, while the GHZ state will be reduced to a product of three or two qubits. Experimentally, the interference of photons generated by independent down-conversion process enabled the first demonstrations of a three-photon GHZ state [9] and a four-photon GHZ state [10]. So far, several schemes for the preparation of the three-particle *W* state [11] and four-particle *W* state [12] have been proposed. Quite recently, the three-photon *W* state has been observed in an experiment [13,14].

In this Brief Report, we propose a feasible scheme by current technologies to generate a four-photon polarization-entangled *W* state which is represented as

$$|W^{(4)}\rangle = \frac{1}{2}(|HHHV\rangle + |HHVH\rangle + |HVHH\rangle + |VHHH\rangle). \qquad (3)$$

We show that this four-photon *W* state can be directly generated by post-selection from a collinear degenerate type-I two-crystal geometry, which has been used to generate a high-intensity entangled state of two photons [15]. The proposed scheme is composed of linear optical elements and photon detectors; fragile interferometric setups are not required. Comparing with the scheme presented in Ref. [12], our scheme does not require single-photon sources.

The proposed setup is shown in Fig. 1. A pulsed laser is used to pump two identically cut type-I crystals with their optic axes aligned in mutually perpendicular planes. After the crystals, a delay line (birefringent element) [15] is used to compensate the time difference between the down-conversion photons created in the first crystal and the second crystal. For a strong pulsed pump, there is a reasonable possibility of simultaneously producing four photons (two down-converted photon pairs) in one single pulse [16,17]. The four-photon state from this parametric down-conversion process consists of three parts: the first one (part A) is two down-conversion photon pairs both coming from the first crystal with horizontal polarization, the second one is (part B) two pairs both coming from the second crystal with vertical polarization, and the third one (part C) is one pair coming from the first crystal with horizontal polarization and the other one coming from the second crystal with vertical polarization. To observe the four-photon polarization-entangled state, the photons should be detected behind narrow band filters, resulting a coherence time longer than the pump pulse
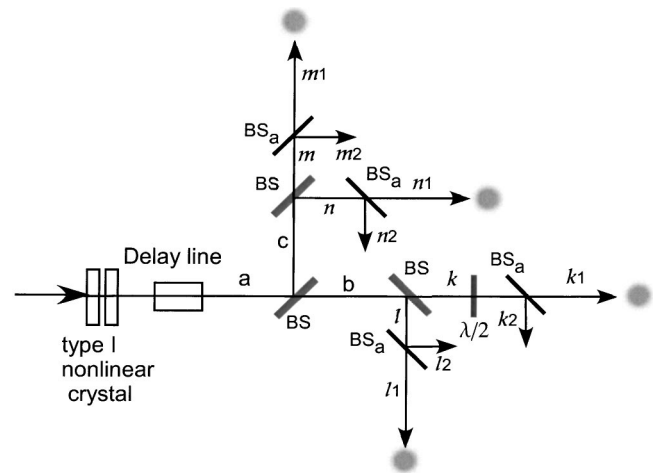

FIG. 1. Schematic representation of the setup for generating the four-photon polarization-entangled *W* state from a two-crystal geometry. BS, 50-50 nonpolarization beam splitter; BS$_a$, adjusting beam splitter.

duration [17,18]. We assume that the probability of creating one horizontal (vertical) photon pair in a single pulse is $P_H(P_V)$. For part C, the probability of simultaneously creating one pair from each crystal is $P_H P_V$ (because the creation of the two pairs are independent). Whereas for part A (part B), the probability of simultaneously creating two pairs from one crystal is $P_H^2(P_V^2)$ due to the stimulation emission process [19]. Now the simplified version of the four-photon state can be written as [18]

$$|\Psi^{(4)}\rangle = P_H|4H_a\rangle + P_V|4V_a\rangle + \sqrt{P_H P_V}|2H_a, 2V_a\rangle. \quad (4)$$

Here $4H_a$ means four horizontal polarization photons in mode a, etc. By setting the polarization of pump beam to θ (relative to horizontal direction) with $\theta = \arctan\sqrt{6}$, we have $P_V = 6P_H$; then, Eq. (4) becomes

$$|\Psi^{(4)}\rangle = P_H(|4H_a\rangle + 6|4V_a\rangle + \sqrt{6}|2H_a, 2V_a\rangle). \quad (5)$$

The four-photon state can also be written as

$$\frac{P_H}{2\sqrt{6}}[(\hat{a}_H^\dagger)^4 + 6(\hat{a}_V^\dagger)^4 + 6(\hat{a}_H^\dagger)^2(\hat{a}_V^\dagger)^2]|0\rangle. \quad (6)$$

Next mode a is directed to a 50-50 nonpolarization beam splitter (BS). The transformation of the BS is assumed to be $\hat{a} \to (1/\sqrt{2})(\hat{b} + \hat{c})$, where $\hat{b}$ and $\hat{c}$ denoting the transmitted mode and reflected mode, respectively. Keeping only those terms which contain two photons in each of the beams, this transforms Eq. (6) into

$$\frac{3P_H}{4\sqrt{6}}[(\hat{b}_H^\dagger)^2(\hat{c}_H^\dagger)^2 + 6(\hat{b}_V^\dagger)^2(\hat{c}_V^\dagger)^2 + (\hat{b}_H^\dagger)^2(\hat{c}_V^\dagger)^2 + (\hat{b}_V^\dagger)^2(\hat{c}_H^\dagger)^2$$
$$+ 4\hat{b}_H^\dagger \hat{b}_V^\dagger \hat{c}_H^\dagger \hat{c}_V^\dagger]|0\rangle. \quad (7)$$

After this mode b and mode c are directed to another 50-50 nonpolarization beam splitter, respectively. Again we assume that at the beam splitter b is transformed into $\hat{b} \to (1/\sqrt{2})(\hat{k} + \hat{l})$ and c is transformed into $\hat{c} \to (1/\sqrt{2})(\hat{m} + \hat{n})$, where $\hat{k}$, $\hat{m}$, $\hat{l}$, and $\hat{n}$ denote the transmitted mode and reflected mode, respectively. After this transform, mode k passes through a half wave plate which rotates its polarization by 90°—namely, $\hat{k}_H \to \hat{k}_V$ and $\hat{k}_V \to \hat{k}_H$. After the BS and the half wave plate and keeping only those terms which contain one photon in each of the four beams, the state can be given by

$$\frac{3P_H}{4\sqrt{6}}[(\hat{k}_V^\dagger \hat{l}_H^\dagger \hat{m}_H^\dagger \hat{n}_H^\dagger + \hat{k}_H^\dagger \hat{l}_V^\dagger \hat{m}_H^\dagger \hat{n}_H^\dagger + \hat{k}_H^\dagger \hat{l}_H^\dagger \hat{m}_V^\dagger \hat{n}_H^\dagger + \hat{k}_H^\dagger \hat{l}_H^\dagger \hat{m}_H^\dagger \hat{n}_V^\dagger)$$
$$+ 6\hat{k}_H^\dagger \hat{l}_V^\dagger \hat{m}_H^\dagger \hat{n}_V^\dagger + \hat{k}_V^\dagger \hat{l}_H^\dagger \hat{m}_V^\dagger \hat{n}_V^\dagger + \hat{k}_V^\dagger \hat{l}_V^\dagger \hat{m}_H^\dagger \hat{n}_V^\dagger + \hat{k}_V^\dagger \hat{l}_V^\dagger \hat{m}_V^\dagger \hat{n}_H^\dagger]|0\rangle. \quad (8)$$

The first term in Eq. (8) represents a four-photon polarization entangled W state which is mixed with other four-photon terms. To observe the four-photon polarization-entangled W state, other terms in Eq. (8) must be eliminated or attenuated greatly. This can be done by directing each of

the four modes (k, l, m, and n) to an adjusting beam splitter (BS_a), respectively, where the transformation of the BS_a is defined by

$$\hat{A}_H \to t_H \hat{A}1_H + \sqrt{1 - t_H^2} \hat{A}2_H,$$

$$\hat{A}_V \to t_V \hat{A}1_V + \sqrt{1 - t_V^2} \hat{A}2_V. \quad (9)$$

Here, $A = k, l, m, n$, and t_H and t_V are amplitude transmission coefficients for the horizontal polarization beam and vertical polarization beam of the BS_a, respectively. It is assumed that $t_V \ll t_H$. Considering only those terms which can result in a fourfold coincidence event of models $k1$, $l1$, $m1$, and $n1$, the final state of the system can be written as

$$\frac{3P_H t_V t_H^3}{4\sqrt{6}}\left[(\hat{k}1_V^\dagger \hat{l}1_H^\dagger \hat{m}1_H^\dagger \hat{n}1_H^\dagger + \hat{k}1_H^\dagger \hat{l}1_V^\dagger \hat{m}1_H^\dagger \hat{n}1_H^\dagger\right.$$
$$+ \hat{k}1_H^\dagger \hat{l}1_H^\dagger \hat{m}1_V^\dagger \hat{n}1_H^\dagger + \hat{k}1_H^\dagger \hat{l}1_H^\dagger \hat{m}1_H^\dagger \hat{n}1_V^\dagger)$$
$$+ \frac{t_V^2}{t_H^2}(6\hat{k}1_H^\dagger \hat{l}1_V^\dagger \hat{m}1_H^\dagger \hat{n}1_V^\dagger + \hat{k}1_V^\dagger \hat{l}1_H^\dagger \hat{m}1_V^\dagger \hat{n}1_V^\dagger + \hat{k}1_V^\dagger \hat{l}1_V^\dagger \hat{m}1_H^\dagger \hat{n}1_V^\dagger$$
$$\left.+ \hat{k}1_V^\dagger \hat{l}1_V^\dagger \hat{m}1_V^\dagger \hat{n}1_H^\dagger)\right]|0\rangle. \quad (10)$$

Equation (10) can be rewritten as

$$\frac{3P_H t_V t_H^3 \sqrt{4 + 39 t_V^4/t_H^4}}{4\sqrt{6}}|\Phi^{out}\rangle, \quad (11)$$

where

$$|\Phi^{out}\rangle = \frac{1}{\sqrt{4 + 39 t_V^4/t_H^4}}\left[2|W^{(4)}\rangle_{k1l1m1n1} + \frac{t_V^2}{t_H^2}(6|HVVV\rangle_{k1l1m1n1}\right.$$
$$+ |VHVV\rangle_{k1l1m1n1} + |VVHV\rangle_{k1l1m1n1}$$
$$\left.+ |VVVH\rangle_{k1l1m1n1})\right]. \quad (12)$$

Here state $|\Phi^{out}\rangle$ has been normalized. Now the fidelity of the system can be given by

$$F = \langle W^{(4)}|\rho^{out}|W^{(4)}\rangle, \quad (13)$$

where $\rho^{out} = |\Phi^{(out)}\rangle\langle\Phi^{(out)}|$. By combining Eqs. (12) and (13), the fidelity can be written as

$$F = \frac{4}{4 + 39 t_V^4/t_H^4}. \quad (14)$$

If we set $t_V^2/t_H^2 \le 1/25$, then we can make the state $|W^{(4)}\rangle$ with a purity larger than 98.5%, once the four-coincidence event is observed.

To obtain the efficiency of generating the four-photon polarization-entangled W state in our scheme, we need calculate the probability of producing a fourfold coincidence event which is given as follows [referring to Eq. (11)]:

$$\eta = \left(\frac{3P_H t_V t_H^3 \sqrt{4 + 39 t_V^4/t_H^4}}{4\sqrt{6}}\right)^2. \quad (15)$$

Given the fidelity of 98.5%, to maximize the efficiency, the parameters in Eq. (15) can be selected as $t_V = 1/5$, $t_H = 1$. Therefore the maximum efficiency is $\eta \approx 1.52 \times 10^{-2} P_H^2$. Assuming that the probability of creating one photon pair in a single pump pulse would be of the order of $P_H \approx 4 \times 10^{-4}$, it is easy to calculate that the probability of producing a four-fold coincidence event in the proposed scheme is $\eta \approx 2.43 \times 10^{-9}$. The generation rate for the four-photon polarization-entangled W state should be 2.43×10^{-1} per second for a pump laser with a 100-MHz repetition rate. Considering the 50% single-photon detection efficiency, the real count rate for the four-photon polarization-entangled W state will be reduced to the order of 1 per minute. In practice, to implement our scheme experimentally, we should estimate the contribution from the next-order term (three-photon pair term ϕ). First, the final output state was calculated to be ϕ' after ϕ passes through three beam splitters, a half wave plate, and four adjusted beam splitters; then, all the relevant terms in ϕ' which can result in a fourfold coincidence event were picked out and used to calculate the total fourfold coinci-dence probability, and at last the probability of producing a fourfold coincidence by ϕ' is calculated to be $\eta' \approx 8.22 \times 10^{-1} P_H^3$. When considering the 50% single-photon detection efficiency, η' will be reduced to $\approx 6.71 \times 10^{-2} P_H^3$. The errors caused by this three-photon pair term are $\eta'/\eta \approx 70.6 P_H$, given $P_H \approx 4 \times 10^{-4}$; the errors will be 2.8%. Because the errors are directly proportional to P_H, the improvement of the count rate of four-photon polarization-entangled W state will be limited if we only increase the pump power. This problem can be solved partly by using a multigigahertz mode-locked laser; then, the count rate can be improved by an order or two of magnitude [20].

In conclusion, we have proposed an experimental scheme for producing four-photon polarization-entangled W state directly from the parametric down-conversion of a two-crystal geometry. In our scheme, only linear optical elements and photon detectors are required. Sophisticated and fragile inter-ferometric setups are unnecessary. Furthermore, this scheme is feasible by current technologies.

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